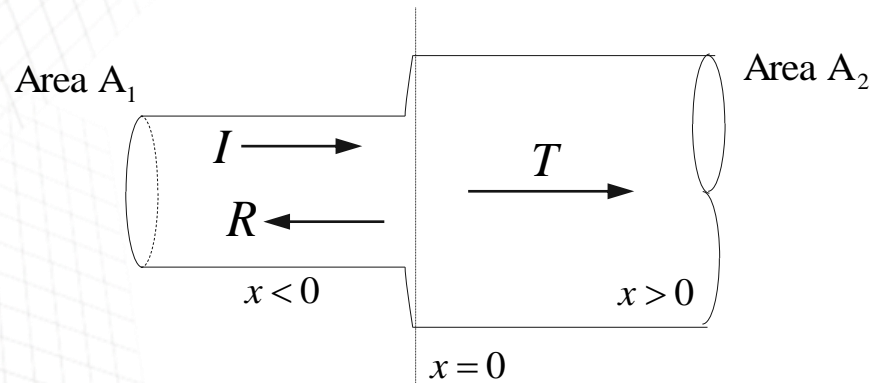


## ❖ Duct Acoustics

### ● Plane wave

- A sound propagation in pipes with different cross-sectional area
- If the wavelength of sound is large in comparison with the diameter of the pipe the sound propagates as an one-dimensional wave ( $\lambda \gg d \rightarrow 1\text{-d wave}$ )



$$p' = I e^{i\omega(t-x/c)} + R e^{i\omega(t+x/c)} \quad \text{in } x < 0$$

$$= T e^{i\omega(t-x/c)}$$

$$\text{in } x > 0$$

$I$ : 입사,  $R$ : 반사,  $T$ : 투과

## ❖ Duct Acoustics

- The mass flux into the junction must equal the mass flux out

$$\rho_0 A_1 u_1 = \rho_0 A_2 u_2$$

- The velocity must equal at both sides of the junction

$$\frac{A_1}{\rho_0 c} (I - R) = \frac{A_2}{\rho_0 c} T$$

- Energy flux)<sub>in</sub> = Energy flux)<sub>out</sub>

$$A_1 p_1' u_1 = A_2 p_2' u_2$$

- The pressure of both sides of junction is continuous

$$I + R = T$$

## ❖ Duct Acoustics

- The amplitudes of other wave,  $R$  and  $T$ , are can be solve from above the relations

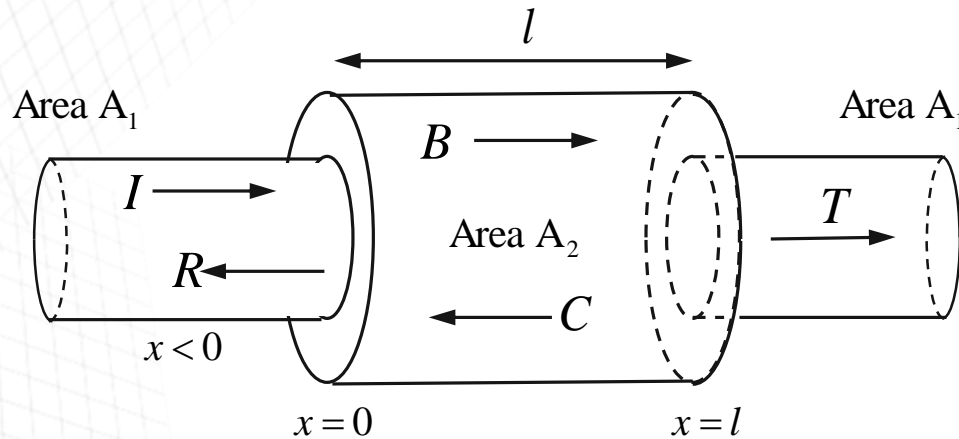
$$R = \frac{A_1 - A_2}{A_1 + A_2} I, \quad T = \frac{2A_1}{A_1 + A_2} I$$

- The transmission loss,  $L_T$  is symmetric in  $A_1$  and  $A_2$

$$\begin{aligned} L_T &= 10 \log_{10} \left( \frac{\text{incident power}}{\text{transmitted power}} \right) \\ &= 10 \log_{10} \left( \frac{A_1 I^2}{A_2 T^2} \right) = 10 \log_{10} \left( \frac{(A_1 + A_2)^2}{4A_1 A_2} \right) \end{aligned}$$

## ❖ Duct Acoustics

- A single expansion-chamber ‘silencer’
  - The simple muffler that is used in car ‘silencer’ consists of inlet and outlet pipes with cross-sectional area  $A_1$ , and expansion chamber between them of cross-sectional area  $A_2$  and length  $l$



## ❖ Duct Acoustics

- The first area change occurs at  $x=0$  and the second occurs at  $x=l$ .

$$\begin{aligned} p' &= I e^{i\omega(t-x/c)} + R e^{i\omega(t+x/c)} && \text{in } x < 0 \\ &= B e^{i\omega(t-x/c)} + C e^{i\omega(t+x/c)} && \text{in } 0 < x < l \\ &= T e^{i\omega(t-x/c)} && \text{in } l < x \end{aligned}$$

- The condition of continuity of mass flux,

$$\begin{aligned} A_1(I - R) &= A_2(B - C) && \text{at } x = 0 \\ A_1 T e^{-i\omega l/c} &= A_2(B e^{-i\omega l/c} - C e^{-i\omega l/c}) && \text{at } x = l \end{aligned}$$

- The condition of continuity of pressure

$$\begin{aligned} I + R &= B + C && \text{at } x = 0 \\ T e^{-i\omega l/c} &= B e^{-i\omega l/c} + C e^{-i\omega l/c} && \text{at } x = l \end{aligned}$$



## ❖ Duct Acoustics

- The algebraic equation when solved for  $R$  and  $T$

$$R = \frac{\left(\frac{A_1}{A_2} - \frac{A_2}{A_1}\right) I i \sin \frac{\omega l}{c}}{2 \cos \omega l / c + i \left(\frac{A_1}{A_2} + \frac{A_2}{A_1}\right) \sin \frac{\omega l}{c}} \quad T = \frac{2 I e^{i\omega l / c}}{2 \cos \omega l / c + i \left(\frac{A_1}{A_2} + \frac{A_2}{A_1}\right) \sin \frac{\omega l}{c}}$$

- However, the simple **'silencer'** does not reduce the total energy of sound in the system.

$$|R|^2 + |T|^2 = |I|^2$$

- Reducing the acoustic energy of transmitted wave  
→ Increasing in the reflected wave
- Sound absorbing material  
→ reduce the acoustic energy by converting it into heat or vibration

## ❖ Duct Acoustics

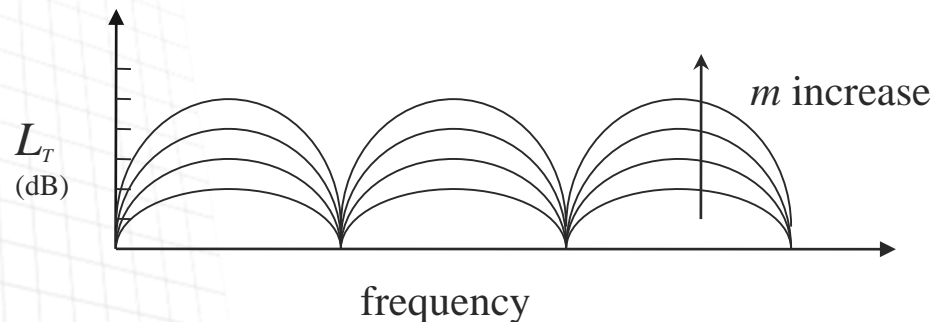
- The transmission loss,  $L_T$  is  $L_T = 10 \log_{10} \left( \frac{|I|^2}{|T|^2} \right)$

$$L_T = 10 \log_{10} \left[ 1 + \frac{1}{4} \left( \frac{A_1}{A_2} - \frac{A_2}{A_1} \right)^2 \sin^2 \left( \frac{\omega l}{c} \right) \right]$$

- The transmission loss is maximum at frequencies for which

$$\sin \omega l / c = \pm 1 \quad \text{i.e.} \quad \omega = \frac{\pi c}{2l}, \frac{3\pi c}{2l}, \frac{5\pi c}{2l} \dots \text{etc}$$

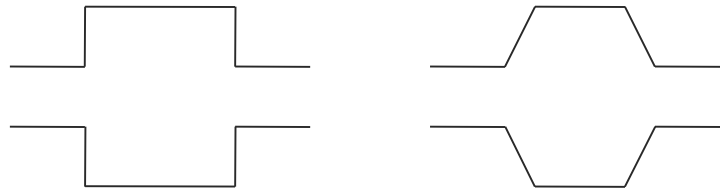
- The effect of expansion ratio  $m = \frac{A_2}{A_1}$



## ❖ Duct Acoustics

### ● Note

- ‘Tuning’ for dominant frequencies of noise
- Theory work for only  $\lambda \gg d$  “Low frequency wave only”
- High frequency waves behave like 3-D
- Also, the geometrical shape of the duct is not important (provided the area change occurs in a distance short in comparison with the wavelength)

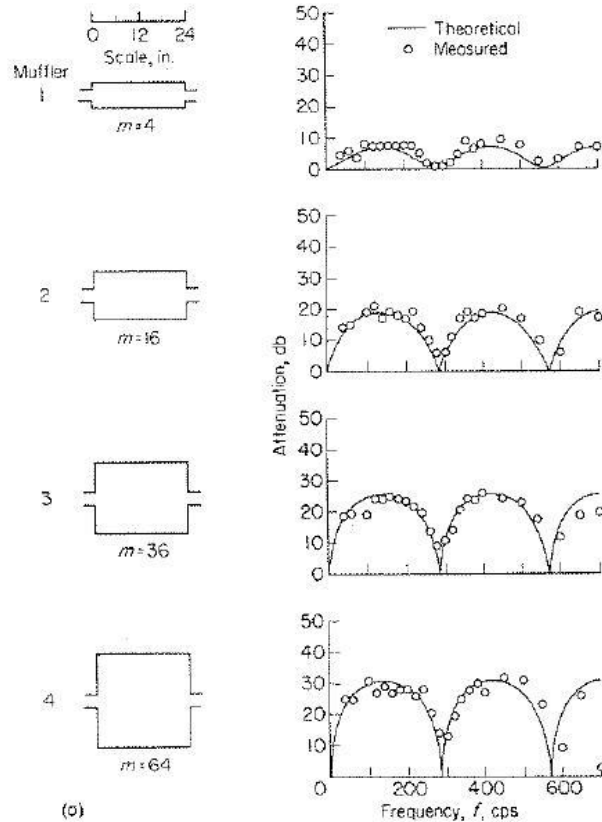




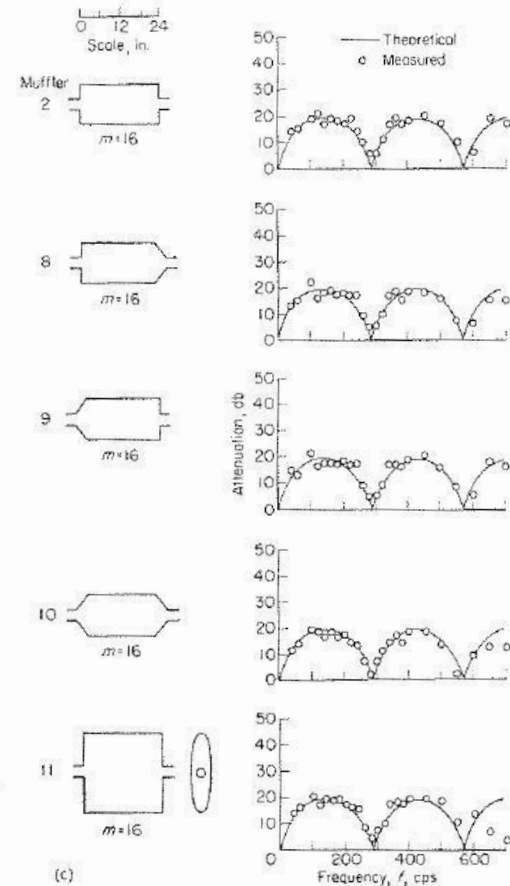
# Transmission & Reflection of Plane Waves

## ❖ Duct Acoustics

Ref. "Theoretical and experimental investigation of mufflers with comments on engine-exhaust muffler design", Davis et al. NACA 1192(1954)

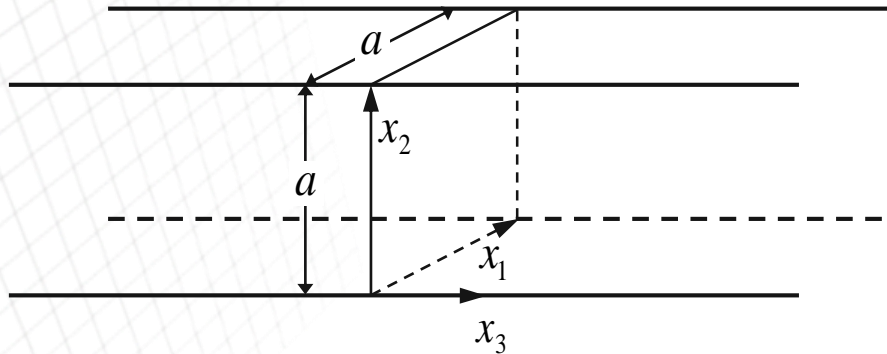


Effect of expansion chamber ratio



Effect of expansion chamber shape

## ❖ Higher order modes



- As an illustration, the sound of frequency  $\omega$  in a rigid walled duct of square cross-section with sides of length  $a$  is considered

$$p'(\mathbf{x}, t) = f(x_1)g(x_2)h(x_3)e^{i\omega t}$$

- With substitution for  $p'$  into the wave equation,

$$\frac{f''}{f} = -\frac{g''}{g} - \frac{h''}{h} - \frac{\omega^2}{c^2} = -\alpha^2$$

## ❖ Higher order modes

- Since a wall boundary condition is applied, function  $f$  is derived like this

$$f(x_1) = A_1 \cos\left(\frac{m\pi x_1}{a}\right), \quad \text{for some integer } m$$

- Similarly function  $g$  is derived like this

$$g(x_2) = A_2 \cos\left(\frac{n\pi x_2}{a}\right), \quad \text{for some integer } n$$

- Finally, function  $h$  is derived to the propagation form

$$h(x_3) = A_{mn} e^{-ik_{mn}x_3} + B_{mn} e^{ik_{mn}x_3} \quad k_{mn} = \sqrt{\frac{\omega^2}{c^2} - \frac{\pi^2}{a^2}(m^2 + n^2)}$$

- The axial phase speed,  $c_p = \omega/k_{mn}$  is now a function of the mode number and the propagation of a group of waves will cause them to disperse.

## ❖ Higher order modes

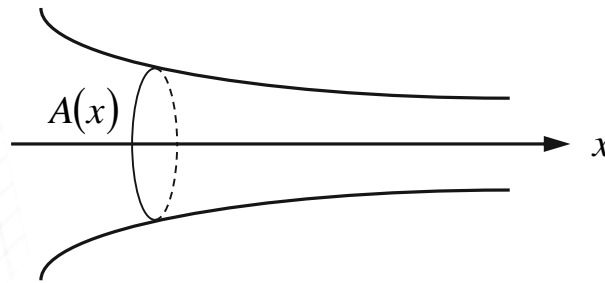
- The pressure perturbation in the  $(m,n)$  mode has the form

$$p'(\mathbf{x}, t) = \cos\left(\frac{m\pi x_1}{a}\right) \cos\left(\frac{n\pi x_2}{a}\right) \left[ A_{mn} e^{-ik_{mn}x_3} + B_{mn} e^{ik_{mn}x_3} \right] e^{i\omega t}$$

- When  $k_{mn}$  is real, the pressure perturbation equation represents that waves are propagating down the  $x_3$  axis with phase speed.
- When  $k_{mn}$  is purely imaginary, i.e. exceeds the cut-off frequency, the strength of mode varies exponentially with distance along the pipe. Such disturbances are evanescent

## ❖ Pipes of varying cross-section

- Wave equation



- If the pipe diameter is small in comparison with both the acoustic wavelength and the length scale over which the cross-sectional area change, most particle motions are longitudinal.

- Conservation of mass 
$$A \frac{\partial \rho}{\partial t} = -\rho_0 \frac{\partial}{\partial x} (uA)$$

- Linearized momentum equation is 
$$\rho_0 \frac{\partial u}{\partial t} = -\frac{\partial p'}{\partial x}$$

- Modified wave equation 
$$\frac{A}{c^2} \frac{\partial^2 p'}{\partial t^2} = \frac{\partial}{\partial x} \left( A \frac{\partial p'}{\partial x} \right)$$



## ❖ Pipes of varying cross-section

### ● Application to the ‘exponential horn’

- Evaluation of the case of ‘exponential horn’ which cross-sectional area defined as,  $A(x)=A_0e^{\alpha x}$
- For such an area variation of wave equation simplifies to

$$\frac{1}{c^2} \frac{\partial^2 p'}{\partial t^2} = \frac{\partial^2 p'}{\partial x^2} + \alpha \frac{\partial p'}{\partial x}$$

- The pressure perturbation in sound waves of frequency  $\omega$  then has the form

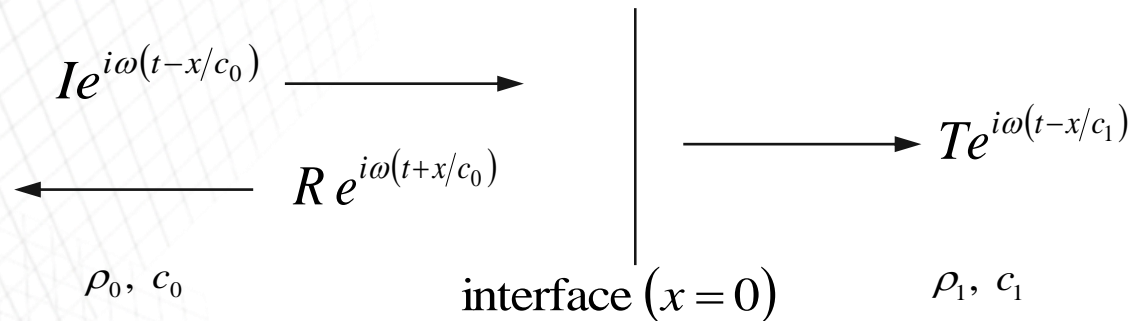
$$p'(x,t) = e^{-\alpha x/2} \{ A e^{i(\omega t - kx)} + B e^{i(\omega t + kx)} \}$$

- Disturbance with  $\omega > \alpha c/2$  propagates and the pressure but not the energy flux attenuates during propagation, while lower frequency modes are ‘cut-off’

## ❖ Normal transmission

### ● Physics at the interface

- When a sound wave crosses an interface between two different fluids some of the acoustic energy is usually reflected.



- There are two boundary conditions
  - The pressure on the two sides of the boundary must be equal
  - The particle velocities normal to the interface must be equal

$$\lambda = cT = \frac{c}{f} = \frac{2\pi c}{\omega} \quad \lambda_0 = \frac{2\pi c_0}{\omega} \quad \lambda_1 = \frac{2\pi c_1}{\omega}$$

## ❖ Normal transmission

- The pressure must be equal at the interface :  $I+R=T$
- The particle velocities normal to the interface must be equal

$$\frac{I}{\rho_0 c_0} - \frac{R}{\rho_0 c_0} = \frac{T}{\rho_1 c_1}$$

- The result pressure coefficients,  $R$  and  $T$ , are determined with  $I$

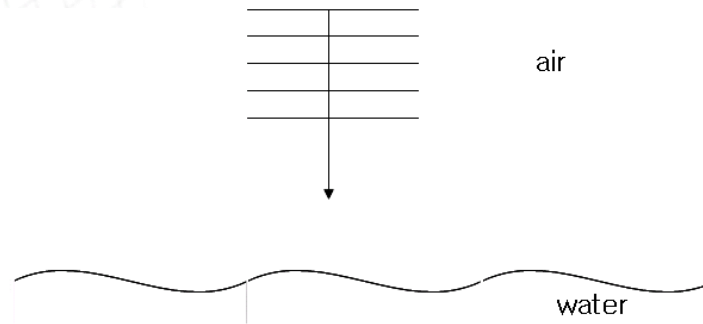
$$R = \left\{ \frac{\rho_1 c_1 - \rho_0 c_0}{\rho_1 c_1 + \rho_0 c_0} \right\} I \quad T = \left\{ \frac{2\rho_1 c_1}{\rho_1 c_1 + \rho_0 c_0} \right\} I$$

- Velocity Transmission Coefficient :  $\frac{T / \rho_1 c_1}{I / \rho_0 c_0} = \frac{2\rho_0 c_0}{\rho_1 c_1 + \rho_0 c_0}$
- The energy flux of the incident wave per unit cross sectional area is equal to that of the reflected and transmitted waves

$$\frac{R^2}{\rho_0 c_0} + \frac{T^2}{\rho_1 c_1} = \frac{(\rho_1 c_1 - \rho_0 c_0)^2 I^2}{(\rho_1 c_1 + \rho_0 c_0)^2 \rho_0 c_0} + \frac{4\rho_1^2 c_1^2 I^2}{(\rho_1 c_1 + \rho_0 c_0)^2 \rho_1 c_1} = \frac{I^2}{\rho_0 c_0}$$

## ❖ Normal transmission

- Reflection from a high and low impedance fluid



- A typical example is aerial sound waves incident onto a water surface. ( $\rho_0 c_0 \ll \rho_1 c_1$ )

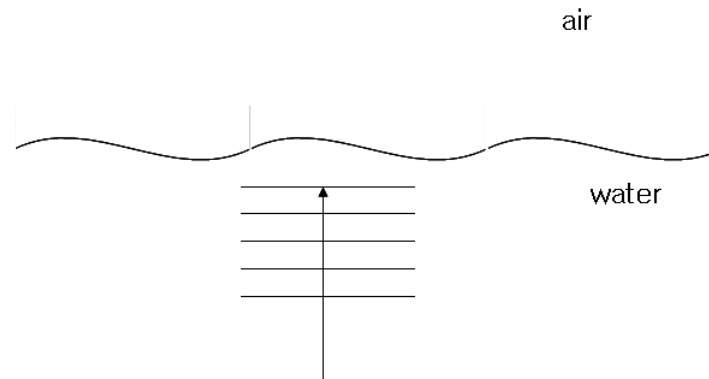
$$R = I, \quad T = 2I$$

- Velocity transmission coefficient  $= \frac{2\rho_0 c_0}{\rho_1 c_1 + \rho_0 c_0} \approx 0$

so, the transmission wave carries negligible energy

## ❖ Normal transmission

- Reflection from a high and low impedance fluid



- In the opposite case, for sound in water incident onto a free surface with air, the reflected and transmitted waves are

$$R = -I, \quad T = 0$$

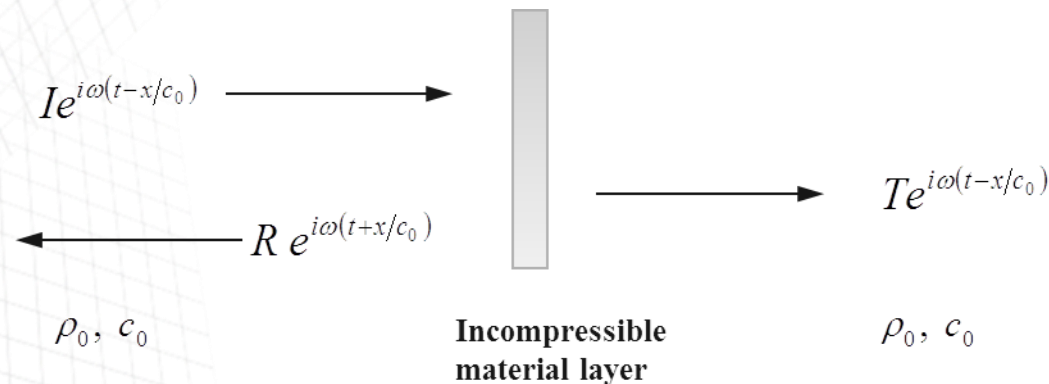
- The acoustic energy is **totally reflected**



## ❖ Sound propagation through walls

### ● Effect of a wall in transmission

- A sound wave normally incident on a plane material layer partitioning a fluid which has uniform acoustic properties,  $\rho_0 c_0$
- Some sound will be reflected from the layer and some will be transmitted through the wall



## ❖ Sound propagation through walls

- There are two boundary conditions that must be satisfied at all times and points
  - The velocity of the wall must be equal to wave of each side
  - A pressure difference across the wall in order to provide the force necessary to accelerate unit area of the surface of material

- By continuity the velocity of wall,

$$u = (I - R) \frac{e^{i\omega t}}{\rho_0 c_0} = \frac{T}{\rho_0 c_0} e^{i\omega t}$$

- The pressure difference is the net force of mass per unit area of the wall

$$\begin{aligned} p_1' &= (I + R)e^{i\omega t} \\ p_2' &= Te^{i\omega t} \end{aligned}$$

$$(I + R - T)e^{i\omega t} = m \frac{\partial u}{\partial t} = m \frac{i\omega T}{\rho_0 c_0} e^{i\omega t}$$

## ❖ Sound propagation through walls

- The result pressure coefficients,  $R$  and  $T$ , are determined with  $I$

$$R = \left\{ \frac{i\omega m}{2\rho_0 c_0 + i\omega m} \right\} I \quad T = \left\{ \frac{2\rho_0 c_0}{2\rho_0 c_0 + i\omega m} \right\} I$$

- Surface Impedance

$$\frac{p_1}{u} = \rho_0 c_0 \frac{(I + R)}{(I - R)} = \rho_0 c_0 \frac{1 + i\omega m}{1 - i\omega m}$$

$$\frac{p_2}{u} = \rho_0 c_0 \frac{(I + R)}{T} = \rho_0 c_0 + i\omega m$$

- Energy transmitted

$$\frac{|T|^2}{\rho_0 c_0} = \frac{4\rho_0^2 c_0^2}{4\rho_0^2 c_0^2 + \omega^2 m^2} \frac{I^2}{\rho_0 c_0}$$

## ❖ Sound propagation through walls

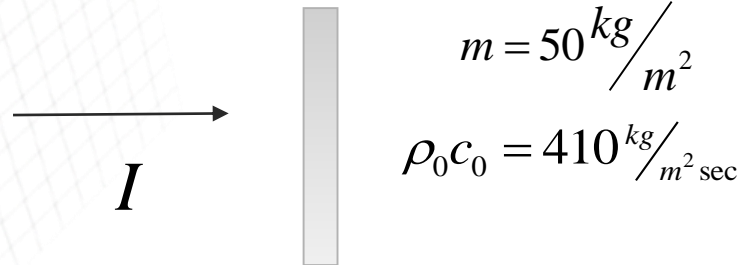
- The transmission loss is dependent on the frequency  $\omega$ .

$$L_T = 10 \log_{10} \left( \frac{4\rho_0^2 c_0^2}{4\rho_0^2 c_0^2 + \omega^2 m^2} \right)^{-1}$$

- For high frequency ( $\omega m \gg \rho_0 c_0$ ), the sound waves mostly reflected
- For low frequency ( $\omega m \ll \rho_0 c_0$ ), the sound waves mostly travels through the wall with very little attenuation
- “Low frequency waves get through a massive wall easily, while high frequency waves are effectively stopped”

## ❖ Sound propagation through walls

- Example) Attenuation by a wall



- Transmission loss 
$$L_T = 10 \log_{10} \left( \frac{|I|^2}{|T|^2} \right) \quad L_T = 10 \log_{10} \left( \frac{4\rho_0^2 c_0^2}{4\rho_0^2 c_0^2 + \omega^2 m^2} \right)^{-1}$$

$$\text{for } f = 10\text{Hz} \quad L_T \approx 12\text{dB}$$

$$\text{for } f = 1\text{kHz} \quad L_T \approx 50\text{dB}$$